Congruence and Similarity

Congruent Triangles: Triangles are congruent when they have exactly the same three sides and exactly the same three angles.

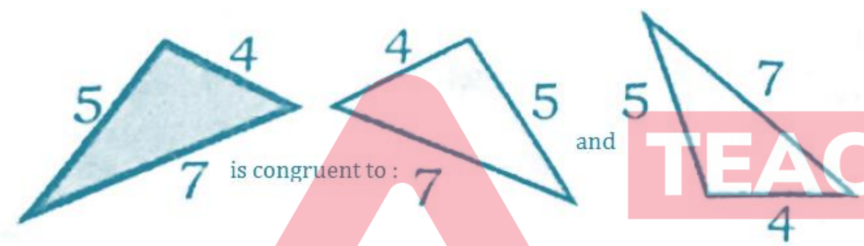
What is "Congruent"?

It means that one shape can become another using turns, flips and/or sliders.

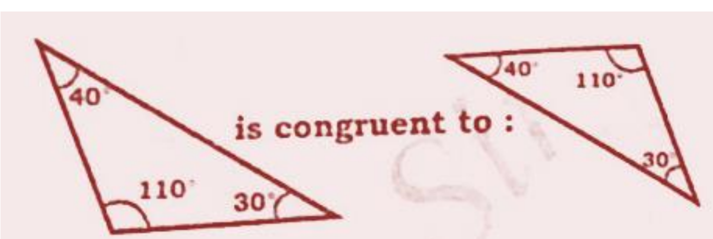
The equal sides and angles may not be in the same position (if there is a turn or a flip), but they are there.

Same Sides: When the sides are same then the triangles are congruent.

For example:

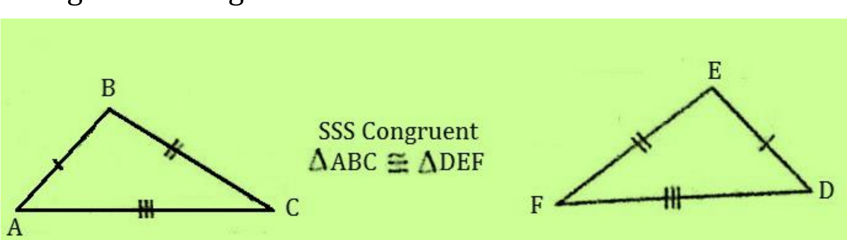


Same Angles: Two triangles with the same angles might be congruent.



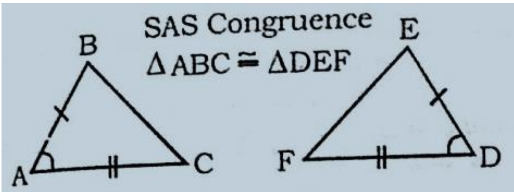
1. SSS congruent

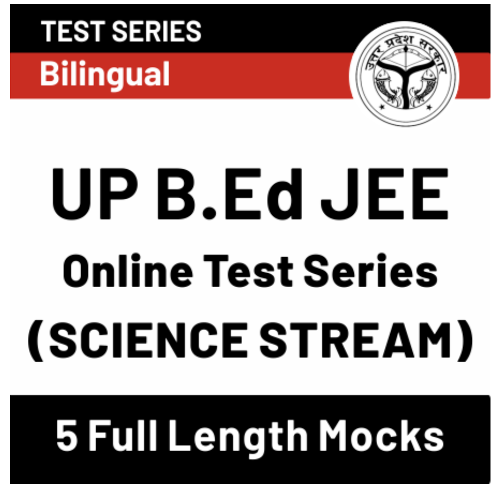
Side – Side – Side congruence. When two triangles have corresponding sides equal that are congruent as shows below, the triangles are congruent.



2. SAS Congruence

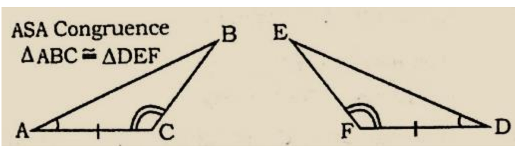
Side – Angle – Side Congruence. When two triangles have corresponding angles and sides equal that are congruent as shown below, the triangles are congruent.





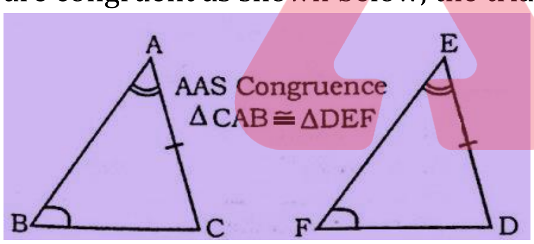
3. ASA Congruence

Angle – Side – Angle Congruence. When two triangles have corresponding angles and sides equal that are congruent as shown below, the triangles themselves are congruent.



4. AAS Congruence OR SAA Congruence →

Angle – Angle – Side congruence. When two triangles have corresponding angles and sides equal that are congruent as shown below, the triangles are congruent.



5. HL Congruence

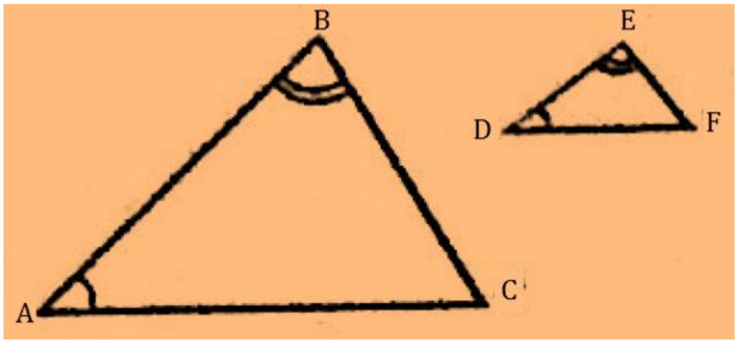
Hypotenuse – leg congruence. When two triangles have corresponding sides equal that are congruent as shown below, the triangles are congruent.

Similarity of Triangles

Two triangles are similar if and only if the corresponding sides are in proportion and the corresponding angles are congruent.

Methods of proving triangles similar:

- If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.

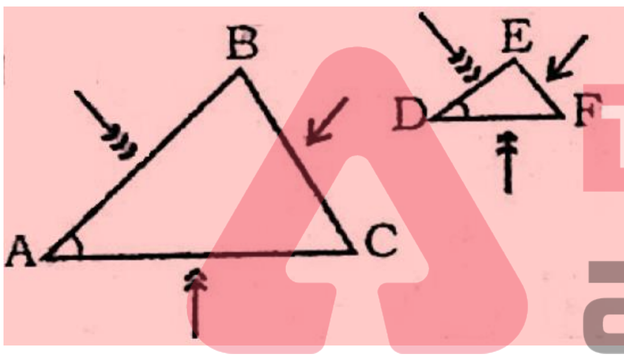


If ∠ A = ∠ D

∠ B = ∠ E

Then : Δ ABC ~ Δ DEF

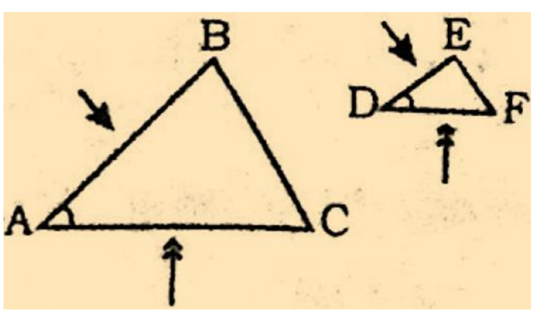
- If the three sets of corresponding sides of two triangle are in proportion, the triangles are similar.



If $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Then : Δ ABC ~ Δ DEF

- If an angle of one triangle is congruent to the corresponding angle of another triangle and the lengths of the sides including these angles are in proportion, the triangles are similar.

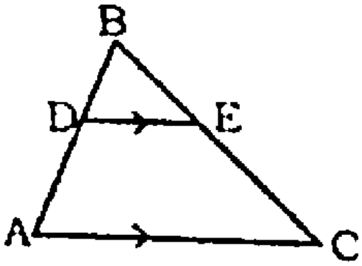


If ∠ A = ∠ D

$\frac{AB}{DE} = \frac{AC}{DF}$

Then : Δ ABC ~ Δ DEF

- If a line is parallel to one side of a triangle and intersects the other two sides of the triangle, the line divides these two sides proportionally.



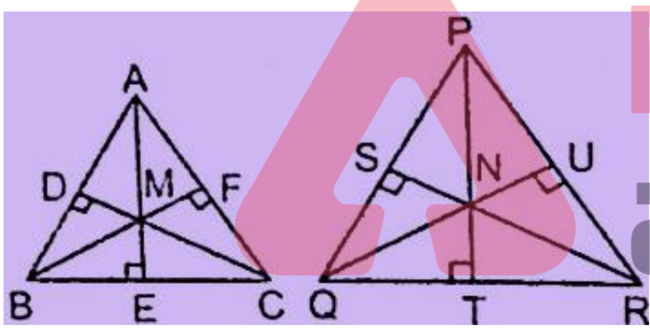
If : $\overline{DE} \parallel \overline{AC}$

Then : $\frac{BD}{DA} = \frac{BE}{EC} \neq \frac{DE}{AC}$

$\frac{BD}{BA} = \frac{BE}{BC} = \frac{DE}{AC}$

Properties of Similar triangles

If the two triangles are similar, then for the proportional / corresponding sides have the following results.



1. {Ratio of sides} = {Ratio of height (altiudes)}

= Ratio of medians

= Ratio of angles bisectors

= Ratio of inradii

= Ratio of circumradii

2. Ratio of areas = Ratio of squares of corresponding sides.

i.e. If Δ ABC ~ Δ PQR,

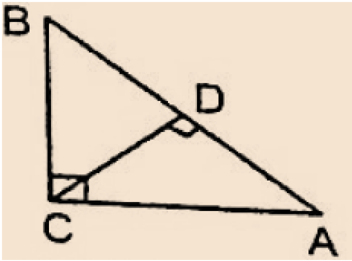
Then,

$$\frac{\text{Area}(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

Perimeter of triangle is : -

$$\frac{\text{Perimeter }(\Delta ABC)}{\text{Perimeter }(\Delta ABC)} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

3. In a right-angled triangle, the triangles on each side of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to the original triangle and to each other too.

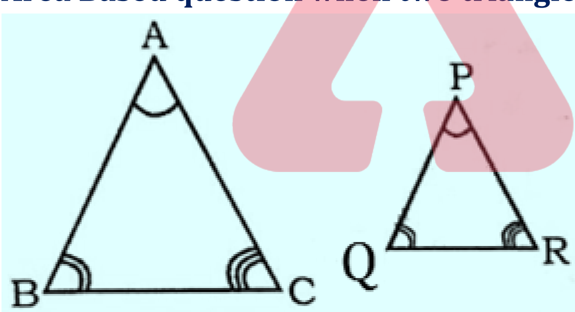


i.e., Δ ABC ~ Δ BDC ~Δ CDA.

Some facts on Right angle triangle

(A) $CD^2 = BD \times DA$
(B) $BC \times CA = BA \times CD$
(C) $BC^2 = BD \times BA$
(D) $AC^2 = AD \times BA$
(E) $\frac{BD}{DA} = \frac{BC^2}{AC^2}$
(F) $\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{CA^2}$

Area Based question when two triangles are similar



If Δ ABC ~ Δ DEF Then

The Ratio of their area is the square of their sides, Medians, Altitudes, Perimeters, Angle Bisectors.

$$\frac{\text{Area}(\Delta ABC)}{\text{Area }(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{AC}{PR}\right)^2 = \left(\frac{BC}{QR}\right)^2$$
$$= \left(\frac{\text{Altitudes}_1}{\text{Altitudes}_2}\right)^2 = \left(\frac{\text{Median}_1}{\text{Median}_2}\right)^2 = \left(\frac{\text{Angle Bisector}_1}{\text{Angle Bisector}_2}\right)^2$$
$$= \left(\frac{\text{Perimeter }\Delta ABC}{\text{Perimeter }\Delta PQR}\right)^2$$