Problem Solving in Mathematics

Mathematics plays a very important role in our day-to-day life. Mathematical skills and its applications form an indispensable tool in our lives, the world over. These days Mathematics is being used more and more in Social Sciences, Management and Commerce in addition to Basic and Life Sciences. Problem solving has, therefore, acquired an important place in the teaching-learning process of mathematics. Besides other approaches, problem solving is also considered as an important approach for teaching mathematics.

What is a Problem?

In general, a problem may be explained as description of an objective not immediately attainable, calling for an action appropriate to attainment of that objective. Suppose a person wants to succeed in a competitive examination, then appropriate actions for the attainment of the objective are:

(i) to look for the syllabus on which the examination is based.

(ii) to analyse the type of questions that are set in the examination.

(iii) to prepare for the examination in the light of the above information etc.

PROBLEM SOLVING IN MATHEMATICS

According to G. Polya year (1981, P.117), a problem is a situation in which a person searches consciously for some appropriate action to attain the clearly, but not easily attainable objective and solving a problem means to find such an action. In mathematics also, problem is a situation for which the child does not have an immediate answer or an obvious mathematical operation or method of finding the answer. Here it must be noted that a problem can be solved in a number of ways and the teacher must encourage the child to follow her own approach(s).

Importance of Problems and Problem Solving in Mathematics

1. Trisecting an angle using ruler and compasses: In this connection, it must be remembered that 'no markings' are allowed on the ruler. This is known as the famous Archimedes problem. With this, he claimed that he could trisect a given angle, if he is allowed to have two markings on the ruler.

2. Squaring a Circle

3. Doubling a Cube

4. Euclid's Fifth Postulate: In attempting to find a proof for it, mathematicians were able to discover Non-Euclidean geometries.

5. The Prime Number Mysteries: Some of the oldest unsolved problems involving prime numbers is the prime number mysteries.
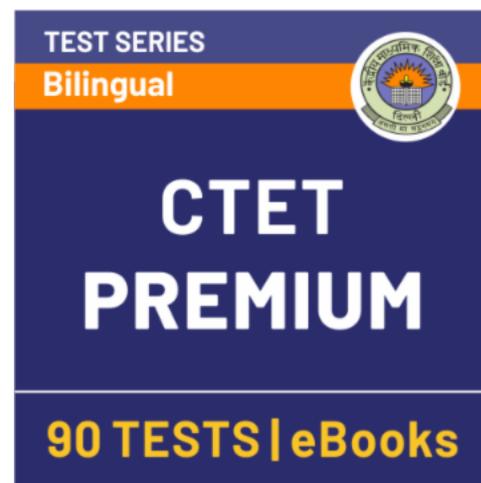
6. Goldbach's Conjecture: 'Is every even number the sum of two primes?' This is another mathematical mystery. In 1742, the German mathematician C. Goldbach wrote a letter to his friend, the great Swiss mathematician Leonhard Euler (1707- 1783), in which he made the conjecture that every even number except 2 was the sum of two primes. This was an interesting statement that was true for every even number he examined, but he could not prove that it was true statement for all even numbers.

7. The Odd Perfect Number Mystery: The ancient Greeks considered some numbers to be perfect. Perfect numbers are numbers which are equal to the sum of their divisors. The number 6 is such a number because 6 = 1+2+3. Another perfect number is 28, since 28 = 1+2+4+7+14. The next perfect number after 28 is 496. Others have been found and all of them are even numbers. No one has ever found an odd perfect number. Also no one has been able to prove that every perfect number must be even.

8. How to Pack Spheres: A geometry problem that is still unsolved involves the packing of spheres such as ping-pong balls. How should spheres be packed in a box so that they use the least possible space? This is similar to a problem of drawing circles in a rectangle.

Problem Solving Approach
Learners learn mathematical thinking most effectively through applying concepts and skills in interesting and realistic contexts which are personally meaningful to them. Thus, Mathematics is best taught by helping learners to solve problems drawn from their own experience. Real-life problems are not always closed, nor do they necessarily have only one solution. The solutions to problems which are worth solving seldom involve only one item of mathematical understanding or only one skill. Rather than learners remembering the single correct method, problem - solving requires them to search the information for clues and to make connections to the various pieces of Mathematics and other knowledge, experiences and skills that they have Strategies for Learning.

According to NCTM (2000) "Problem solving means engaging in a task for which the solution is not known in advance". Any mathematical situation can be a problem for a learner if learner has not previously learned about how to solve that. Once the learner learns how to solve a problem, it becomes an exercise. Teaching through problem solving and teaching problem solving are two different approaches. Teaching problem solving usually works on guess and check, working backward etc. methods.

In teaching through problem solving, teacher will setup the context and explain the problem. Now, learners work on the problem and the teacher monitors their progress. After stipulated time each learner of the class shares his/her ideas with the whole class and then they compare as to which idea is best for solving that particular problem. In this way learner learns many new mathematical ideas and procedures.

Problems of Teaching
1. A sense of fear and failure regarding mathematics among a majority of children,
2. A curriculum that disappoints both a talented minority as well as the nonparticipating majority at the same time,

3. Crude methods of assessment that encourage perception of mathematics as mechanical computation, and

4. Lack of teacher preparation and support in the teaching of mathematics. Other Systemic Problems

One major problem is that of compartmentalisation: there is very little systematic communication between primary school and high school teachers of mathematics, and none at all between high school and college teachers of mathematics. Most school teachers have never even seen, let alone interacted with or consulted, research mathematicians. Those involved in teacher education are again typically outside the realm of college or research mathematics.

curricular acceleration: a generation ago, calculus was first encountered by a student in college. Another generation earlier, analytical geometry was considered college mathematics. But these are all part of school curriculum now.

Gender Issue: Mathematics tends to be regarded as a 'masculine domain'. This perception is aided by the complete lack of references in textbooks to women mathematician.