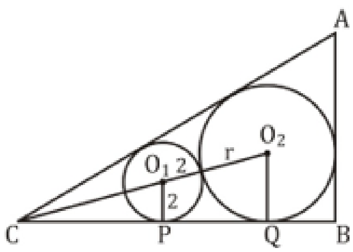
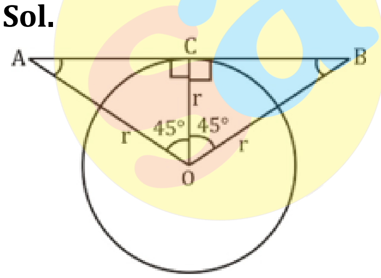
Solutions

S1. Ans.(b)
Sol.



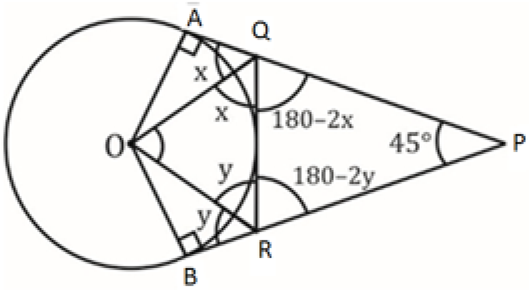
Let the centers are O_1 & O_2
From ΔCO_1P,
$\sin 30° = \frac{O_1P}{O_1C}$
$O_1C = 4$ cm
$CO_2 = 6 + r$
$QO_2 = r$
From $\Delta\ CO_2Q$,
$\sin 30° = \frac{r}{CO_2}$
$CO_2 = 2r$
$2r = 6 + r$
$r = 6$ cm

S2. Ans.(c)
Sol.



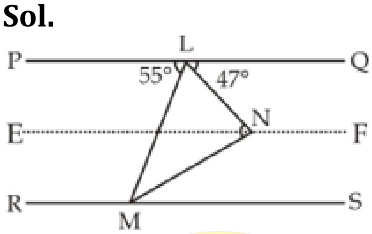
In ΔAOC
$AC = OC = r$
Area of $\Delta AOC = \frac{1}{2} \times r \times r = \frac{r^2}{2}$
Shaded area in AOC
$= \frac{r^2}{2} - \frac{\pi r^2}{360} \times 45°$
$= \frac{4r^2 - \pi r^2}{8}$
$= \frac{4-\pi}{8}$
Required Area $= 2\left(\frac{4-\pi}{8}\right) = \frac{(4-\pi)}{4}$ cm^2

S3. Ans.(a)
Sol.



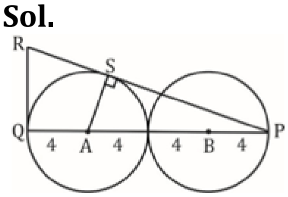
Let
$\angle OQA = \angle OQR = x$
$\angle QRO = \angle ORB = y$
In triangle PRQ
$\angle Q + \angle R + \angle P = 180$
$\Rightarrow 180 - 2y + 180 - 2x + 45 = 180$
$2(x + y) = 225$
$x + y = 112\frac{1}{2}$
$\angle QOR = 180 - (x+y) = 180 - 112\frac{1}{2}$
$\Rightarrow 67\frac{1}{2}°$

S4. Ans.(a)
Sol.



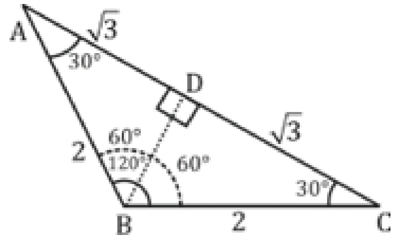
$\Rightarrow \angle PLM = \angle LMS = 55°$ (Alternate angles)
$\rightarrow$ Lets draw EF || PQ || RS
$\Rightarrow \angle QLN = \angle LNE = 47°$ (Alternate angles)
$\therefore \angle ENL + \angle MNE = 67°$
$\Rightarrow \angle MNE = 67° - 47° = 20°$
Similarly, EF || RS
$\therefore \angle ENM = \angle NMS = 20°$ (Alternate angles)

S5. Ans.(c)
Sol.

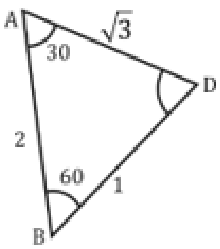


$QP = 4 \times 4 = 16$
$AP = 12, \quad AS = 4$
$PS = \sqrt{(12)^2 - (4)^2}$
$= 8\sqrt{2}$
$\Delta PQR \sim \Delta PSA$
$\frac{RQ}{AS} = \frac{QP}{PS} \Rightarrow \frac{RQ}{4} = \frac{16}{8\sqrt{2}}$
$RQ = 4\sqrt{2}$ cm

S6. Ans. (d)
Sol.

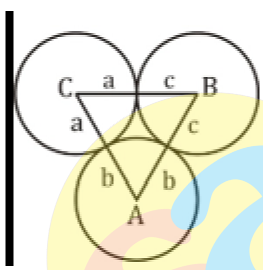


4 : 1 : 1 =
Let Angle= 4x + x + x
ATQ
4x + x + x = 180
x = 30
BD is perpendiculars to side AC
In Δ ABD



Required ratio= $\frac{2\sqrt{3}}{2+2+\sqrt{3}+\sqrt{3}} = \frac{\sqrt{3}}{2+\sqrt{3}}$

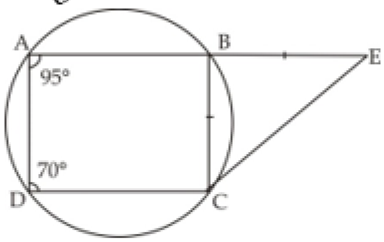
S7. Ans.(d)
Sol.



Semi perimeter of ABC = a + b + c
Required area = $\sqrt{(a+b+c)abc}$ (by hero's formula)

S8. Ans.(b)
Sol.
r = 3, R = 4, D = 10
Direct common Tangent = $\sqrt{(D)^2-(R-r)^2}$
$= \sqrt{100-1} = \sqrt{99}$
Indirect common Tangent = $\sqrt{(D)^2-(R+r)^2} = \sqrt{51}$
DCT : ICT = $\sqrt{99}$: $\sqrt{51}$
$= \sqrt{33}$: $\sqrt{17}$

S9. Ans.(a)
Sol.
ATQ,

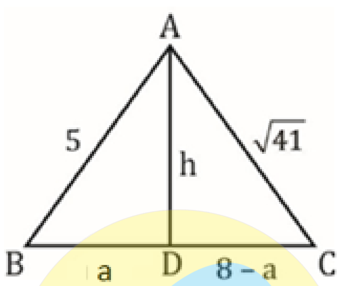


→ In circle quadrilateral, sum of opposite angles is 180°
∴ ∠BCD = 180° - ∠BAD = 180° - 95° = 85°
And,
∠ABC = 180° - ∠ADC = 180° - 70° = 110°
Now,
∠EBC = 180° - ∠ABC = 180° - 110° = 70°
Now,
∵BE = BC

$$\therefore \angle BCE = \angle BEC = \frac{180° - 70°}{2} = 55°$$

∴ ∠DCE = ∠BCE + ∠BCD
= 55° + 85° = 140°

S10. Ans.(b)
Sol.



AS BC=8
LET BD=a then CD=8-a
$(5)^2 - a^2 = (\sqrt{41})^2 - (8-a)^2$
$25 - a^2 = 41 - 64 - a^2 + 16a$
a = 3
So, h = 4 cm
$A = \frac{1}{2} \times 4 \times 3 = 6cm$

[**Hitting method** → ABD is right angle triangle Hypotenuse = 5 so, either base (BD) and perpendicular (Ad) is 3 & 4. Because of Triplets (3, 4, 5). In any case area = ½ × 4 × 3 = 6]

S11. Ans.(a)
Sol.
Centroid of ΔABC coincide with the centroid of triangle formed by mid-points of AB, BC and CA.
∴ Required coordinates

$$\equiv \left(\frac{4+3+2}{3}, \frac{2+3+2}{3}\right) \equiv (3, 7/3)$$

S12. Ans.(d)

Sol.

$a = BC = \sqrt{0^2 + (12-0)^2} = 12$

$b = AC = \sqrt{(0-8)^2 + (6-0)^2} = 10$

$c = AB = \sqrt{8^2 + 6^2} = 10$

Incentre is

$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

$i.e. \left(\frac{12\times0+10\times8+10\times8}{12+10+10}, \frac{12\times6+10\times12+10\times0}{12+10+10}\right)$

$= \left(\frac{160}{32}, \frac{192}{32}\right) = (5, 6)$

S13. Ans.(c)

Sol.

Mid-point of (p, q) and $(q, -p)$ is $\left(\frac{p+q}{2}, \frac{q-p}{2}\right)$,

which is given $\left(\frac{r}{2}, \frac{s}{2}\right)$.

$\therefore \frac{p+q}{2} = \frac{r}{2}$

And $\frac{q-p}{2} = \frac{s}{2}$

Now, length of segment

$= \sqrt{(p-q)^2 + (q+p)^2} = \sqrt{s^2 + r^2}$

S14. Ans.(c)

Sol.

$\Delta = \frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix}$

$= \frac{1}{2} \times (2a) \times a \times \begin{vmatrix} 1 & 0 & 1 \\ t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \end{vmatrix}$

$\therefore \Delta = 0 \Rightarrow (t_1 - t_2) + (t_1^2 t_2 - t_2^2 t_1) = 0$

$\Rightarrow (t_1 - t_2) + t_1 t_2 (t_1 - t_2) = 0$

$\Rightarrow (t_1 - t_2)(1 + t_1 t_2) = 0$

$\Rightarrow t_1 t_2 = -1$

S15. Ans.(c)

Sol.

Let A(a, 0) and B(0, b) be two points on respective coordinate axes and (–5, 4) divides AB in the ratio 1 : 2.

$\therefore -5 = \frac{1\times0+2\times a}{3} \Rightarrow a = \frac{-15}{2}$

and $4 = \frac{1\times b+2\times0}{3} \Rightarrow b = 12$

Hence, equation of line joining

$\left(-\frac{15}{2}, 0\right)$ and $(0, 12)$ is

$(y - 0) = \frac{12-0}{0+\frac{15}{2}} \cdot \left(x + \frac{15}{2}\right)$

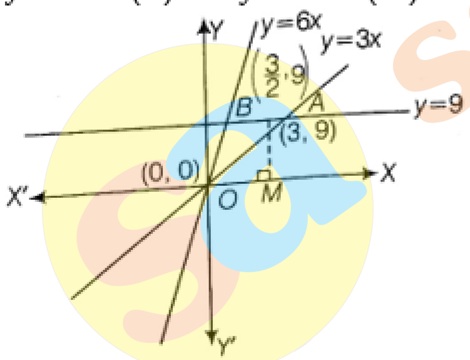
$\Rightarrow 8x - 5y + 60 = 0$

S16. Ans.(c)
Sol.
Let the coordinate of the moving point P be (h, k).
Then, $[h-(m+n)]^2+[k-(n-m)]^2$
$=[h-(m-n)]^2+[k-(n+m)]^2$
$\Rightarrow h^2+(m+n)^2-2h(m+n)+k^2+(n-m)^2-2k(n-m)$
$=h^2+(m-n)^2-2h(m-n)+k^2+(n+m)^2-2k(m+n)$
$\Rightarrow -2[h(m+n)+k(n-m)]=-2[h(m-n)+k(m+n)]$
$\Rightarrow mh+nh+nk-mk=$ mh – nh + mk + nk
⇒ 2nh = 2mk ⇒ nh = mk
∴ Required locus is nx = my

S17. Ans.(b)

S18. Ans.(d)
Sol.
We have, Δ = 3 sq units
$\therefore \left|\frac{1}{2}\begin{vmatrix}5 & 2 & 1\\ 3 & 4 & 1\\ a & 5 & 1\end{vmatrix}\right|=3$
$\left|\frac{1}{2}[5(4-5)-2(3-a)+1(15-4a)]\right|=3$
$\Rightarrow 2-a=\pm 3 \Rightarrow a=5\ or-1$
∴ a = 5

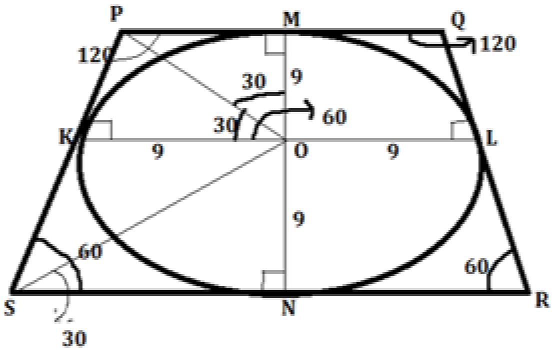
S19. Ans.(a)
Sol.
Given, lines are y = 3x .....(i)
y = 6x ....(ii) and y = 9 ......(iii)



On solving Eqs. (i) and (iii), we get
x = 3, y = 9
∴ Coordinates of A = (3, 9)
On solving Eqs. (ii) and (iii), we get
$x=\frac{3}{2}, y=9$
∴ Coordinates of B $=\left(\frac{3}{2},9\right)$
Area of $\Delta OAB=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 3 & 9 & 1\\ 3/2 & 9 & 1\end{vmatrix}$
$=\left|\frac{1}{2}\left[27-\frac{27}{2}\right]\right|=\frac{27}{4}$ sq units

S20. Ans.(d)
Sol.
Vertices of parallelogram ABCD in order are (1, 2), (4, 3), (6, 6), and (3, 5)
$\therefore$ Area of parallelogram $= \frac{1}{2}\begin{vmatrix}(1-6) & (2-6)\\(4-3) & (3-5)\end{vmatrix}$
$= \frac{1}{2}\begin{vmatrix}-5 & -4\\1 & -2\end{vmatrix} = \frac{14}{2} = 7$ sq units

S21. Ans.(c)
Sol.
Here given a diagram as shown in the question



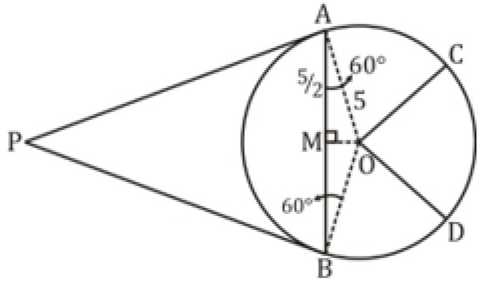
$\angle RSP = \angle SRO = 60$ (Given)
$\angle PQR = \angle QPS = 120$ (Given)
In a quadrilateral PMOK
$\angle MOK = 180 - 120 = 60$
$\angle POM = \angle KOP = 30$
OK = OM = 9 (Given)
In ΔPOM
$\tan 30 = \frac{PM}{9} \Rightarrow PM = 3\sqrt{3} = PK = MQ = QL$

Similarly In ΔSON
$\tan 30 = \frac{ON}{SN}$
$SN = 9\sqrt{3} = SK = NR = RL$
So, perimeter PQRS
$= 3\sqrt{3} \times 4 + 9\sqrt{3} \times 4$
$= 4 \times 12\sqrt{3}$
$= 48\sqrt{3}$

S22. Ans.(c)
Sol.
$\angle BDC = 30°$
$\angle BAC = \angle BDC$ ($\because$ both angle are on same arc)
$\angle BAC = 30°$
$\angle ACB = x° = 180° - (100° + 30°)$
$x° = 50°$

S23. Ans.(b)

Sol.



AB = 5 (Given)
Radius = 5 (given)
Hence ΔAOB is an equilateral triangle,
So,
$\theta = 60° = \angle BOM$
because AO = OB = radius
$\angle AOB = 60$
$\angle PAM = \angle PBM = 90 - 60 = 30$ each
$\angle APB = 180 - (30 + 30) = 120 = \angle COD$
Because PA∥OC & PB∥OD

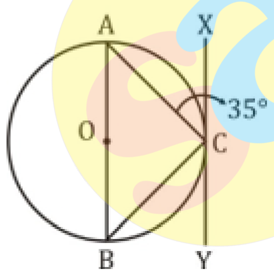
S24. Ans.(a)

Sol.

TQ = TP (∵ Both are tangents on circle from same point)
And also
TR = TP
∴ TQ = TR
⇒ TQ : TR = 1 : 1

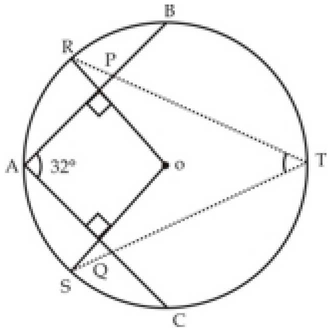
S25. Ans.(C)

Sol.



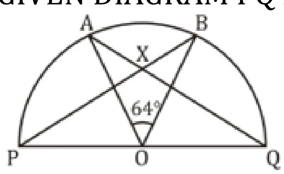
$\angle ACB = 90$ [angle on semicircle is always 90]
$\angle ACX = 35$
$\angle ACX = \angle ABC = 35°$ (it is a property)
So,
$\angle BAC$ or $\angle CAB = (90 - 35) = 55°$

S26. Ans.(b)
Sol.



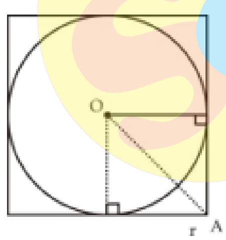
P and Q are mid points of AB and AC so OP and OS will be perpendicular at AB and AC respectively.
$\therefore \angle APO = \angle AQO = 90°$
$\Rightarrow \angle ROS = 180° - 32°$
$\angle ROS = 148°$
$\angle RTS = \frac{1}{2} \angle ROS$
$\angle RTS = 74°$

S27. Ans.(c)
Sol.
GIVEN DIAGRAM PQ is diameter of circle with center O, angle AOB=64



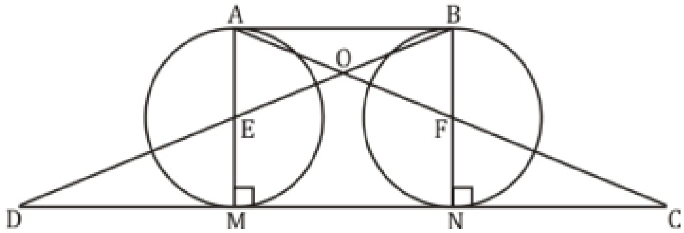
If we draw line AP Then
$\angle APB = ½ \angle AOB = 32$ (Angle at circumference with same arc is always half of the angle at center)
$\angle PAQ = 90$ (angle at semicircle with diameter is always 90)
So,
$\angle AXP = 180 - (\angle APB + \angle PAQ) = 180 - (90+22) = 58°$

S28. Ans.(c)
Sol.



Radius of larger circle = R
Radius of smaller circle = r
Centre of smaller circle = c
$OA = R\sqrt{2}$
$AC = r\sqrt{2}$
$OA = R + r + r\sqrt{2}$
$R\sqrt{2} = R + r(\sqrt{2}+1)$
$R(\sqrt{2}-1) = r(\sqrt{2}+1)$
$r = \left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) R$ or $(3 - 2\sqrt{2})R$

S29. Ans.(b)
Sol.
ACCORDING TO QUESTION FIGURE IS DRAWN, two circles with center E & F



$\angle AEB \approx MED$
$\angle AFB \approx CFN$
So,
$AB = DM = CN$
&
$AB = MN$
[As shown in fig]

$$\frac{AB}{DM + MN + NC} = \frac{AB}{DC} = \frac{1}{3}$$

So,

$$\frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta DOC} = \frac{AB^2}{DC^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

S30. Ans.(a)
Sol.
$\angle ADC = 130°$
$\therefore \angle ABC = 180° - \angle ADC$
$\angle ABC = 50°$
$\angle CAB = 90° - \angle ABC$
$\angle CAB = 90° - 50° = 40°$