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T.B.C. : XZH-S-STSS

Test Booklet Series

D

Serial

1007624

TEST BOOKLET

STATISTICS

Paper II

Time Allowed : Two Hours ***Maximum Marks : 200***

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series Code A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. ***DO NOT*** write ***anything else*** on the Test Booklet.
4. This Test Booklet contains **80** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
5. You have to mark all your responses ***ONLY*** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***only the Answer Sheet***. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**

    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.

    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.

    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. If y_1, y_2, y_3 and y_4 are independent with $E(y_1) = E(y_2) = (\theta_1 + \theta_2)$,
$E(y_3) = E(y_4) = (\theta_1 + \theta_3)$, and
$V(y_i) = \sigma^2$, i = 1, 2, 3 and 4. For estimability of parametric function $\boldsymbol{l'\theta} = l_1\theta_1 + l_2\theta_2 + l_3\theta_3$, consider the following conditions :

1. $l_1 = l_2 + l_3$
2. $l_2 = l_1 + l_3$
3. $l_3 = l_1 + l_2$

Which of the above condition(s) is/are correct ?

(a) 1 only
(b) 2 only
(c) 3 only
(d) 1, 2 and 3

2. Let an $n \times 1$ vector $y \sim N(X\beta, \sigma^2 I)$ where X is an $n \times p$ matrix. Let $H = (h_{ij}) = X(X'X)^{-1}X'$ be an $n \times n$ matrix. Let $e = y - \hat{y}$; $\hat{y} = X\hat{\beta}$ be the $n \times 1$ residual vector where $\hat{\beta}$ is the least squares estimate of β. Then which one of the following is ***not*** correct ?

(a) Rank(H) = p
(b) $0 < h_{ii} < 1; 1 \le i \le n$
(c) $V(\hat{y}) = \sigma^2 H$
(d) $Cov(e_i, e_j) = \sigma^2 h_{ij}$

3. For n observations $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ two models were fitted.

1. $y_i = \alpha + \beta x_i + \text{error}; i = 1, 2, ..., n$
2. $y_i = \alpha + \beta x_i + \gamma x_i^2 + \text{error}; i = 1, 2, ..., n$

Let $\hat{y}_i$ and $\hat{y}_i^*$ be the estimated values of y_i from the two models and if $E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$ and $E^* = \sum_{i=1}^{n} (y_i - \hat{y}_i^*)^2$, then which one of the following is correct ?

(a) $E \ge E^*$
(b) $E \le E^*$
(c) $E = E^*$
(d) It is possible that $E = 0, E^* > 0$

4. Let Y_1, Y_2, Y_3 and Y_4 be uncorrelated observations. It is given that

1. $E(Y_1) = \beta_1 + \beta_2 + \beta_3 = E(Y_2)$
   $E(Y_3) = \beta_1 - \beta_2 = E(Y_4)$
2. $V(Y_i) = \sigma^2; i = 1, 2, 3, 4$

Define $e_1 = \frac{1}{\sqrt{2}}(Y_1 - Y_2)$ and $e_2 = \frac{1}{\sqrt{2}}(Y_3 - Y_4)$. Then an unbiased estimator of σ^2 is given by

(a) $\frac{1}{2}\left(e_1^2 - e_2^2\right)$
(b) $\frac{1}{2}\left(e_1^2 + e_2^2\right)$
(c) $e_1^2 + e_2^2$
(d) $\frac{1}{4}\left(e_1^2 + e_2^2\right)$

5. Consider the following ANOVA table :

Source of variation	Degrees of freedom	Sum of squares	Mean SS	Test statistic
Treatment	w	x	y	5·22
Error	12	z	25	
Total	14	561		

The values of w, x, y and z that complete the above table are respectively

(a) 2, 261, 130·5, 300
(b) 2, 300, 150, 261
(c) 2, 250, 125, 311
(d) 2, 311, 155·5, 250

6. For estimating the mode of a normal distribution, which one of the following is the most efficient estimator ?

(a) Sample mean

(b) Sample median

(c) Sample mode

(d) Largest observation

7. Let $X_1, X_2, X_3, ..., X_n$ be iid random variables with $E(X_i) = \mu$ and $E(X_i^2) < \infty$. Then the consistent estimator for μ is

(a) $\frac{2i^2}{n(n+1)} \sum x_i$

(b) $\frac{2}{n(n+1)} \sum i x_i$

(c) $\frac{2i}{n(n+1)} \sum x_i$

(d) $\frac{2}{n(n+1)} \sum i^2 x_i$

8. If $X_1, X_2, X_3, ..., X_n$ are iid random variables from $N(\mu, \sigma^2)$, then consider the following statements :

1. $\overline{X} = \frac{1}{n} \sum X_i$ is an unbiased and consistent estimator of μ.

2. $S^2 = \frac{1}{n-1} \sum (X_i - \overline{X})^2$ is an unbiased and consistent estimator of σ^2.

3. $S_1^2 = \frac{1}{n} \sum (X_i - \overline{X})^2$ is an unbiased and consistent estimator of σ^2.

Which of the above statements are correct ?

(a) 1 and 2 only

(b) 1 and 3 only

(c) 2 and 3 only

(d) 1, 2 and 3

9. The maximum likelihood estimators (mle) of the parameters α and λ (λ being large) of the distribution $f(x) = \frac{1}{\Gamma(\lambda)} \left(\frac{\lambda}{\alpha}\right)^{\lambda} e^{-\frac{\lambda x}{\alpha}} x^{\lambda - 1}$; $0 < x < \infty, \lambda > 0$ are respectively

[Given that $\frac{\partial}{\partial \lambda} \log \Gamma(\lambda) = \log \lambda - \frac{1}{2\lambda}$]

(a) G and $\frac{1}{2 \log \left(\frac{\overline{x}}{G}\right)}$

(b) $\overline{x}$ and $\frac{1}{2 \log \left(\frac{G}{\overline{x}}\right)}$

(c) $\overline{x}$ and $\frac{1}{2 \log \left(\frac{\overline{x}}{G}\right)}$

(d) G and $\frac{1}{2 \log \left(\frac{G}{\overline{x}}\right)}$

where G is the geometric mean.

10. Let the random variables $X_1, X_2, X_3, ..., X_n$ follow $U(\theta, \theta + 1)$. Then which one of the following statements is ***not*** correct ?

(a) $\left(\overline{X} - \frac{1}{2}\right)$ is an unbiased estimate of θ

(b) UMVUE will not exist for θ

(c) Any value of θ in the interval $\left[X_{(n)}^{-1}, X_{(1)}\right]$ is an mle

(d) $X_{(1)}$ is an mle which is sufficient for θ

11. Let $X_1, X_2, X_3, ..., X_n$ be a random sample from a normal distribution $N(\mu\ \theta)$; where θ is unknown. Then the statistic $T = \sum_{i=1}^{n}(x_i - \mu)^2$ is

(a) sufficient but not complete

(b) complete but not sufficient

(c) both sufficient and complete

(d) neither sufficient nor complete

12. Consider a random sample of size 64 from a normal population with mean μ and standard deviation σ (unknown). The computations obtained from 64 sample observations are $\bar{x} = 60{\cdot}4$ and $\sum_{i=1}^{64}(x_i - \bar{x})^2 = 6300$. The 95% confidence interval for μ is

(a) (58·44, 62·36)

(b) (57·95, 62·85)

(c) (50·4, 70·4)

(d) (58·32, 62·48)

13. Let X be a random variable with $N(\mu, \sigma^2)$, μ and σ^2 are unknown. Then, which one of the following is a simple hypothesis ?

(a) $\mu = \mu_0,\ \sigma^2 \neq \sigma_0^2$

(b) $\mu = \mu_0,\ \sigma^2 = \sigma_0^2$

(c) $\mu > \mu_0,\ \sigma^2 = \sigma_0^2$

(d) $\mu = \mu_0,\ \sigma^2 < \sigma_0^2$

where μ_0 and σ_0^2 are known

14. Let X be a random variable from $U(0, \theta)$. Consider the critical region $\omega = \{x : x > 1\}$ to test $H_0 : \theta = 1$ against $H_1 : \theta = 2$. The size of the test is

(a) 0·05

(b) 0·01

(c) 0

(d) 0·1

15. Let X and Y be two independent random variables with $U(0, \theta)$. We are testing the hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$. The probability of type-I error and the power of the test based on the critical region $\{X + Y > 0{\cdot}75\}$ are respectively

(a) 0·65, 0·85

(b) 0·35, 0·93

(c) 0·72, 0·93

(d) 0·72, 0·85

16. To obtain a critical region (or cut-off point) in testing a statistical hypothesis, we need the distribution of a test statistic

(a) Under H_0

(b) Without any assumption

(c) Under H_1

(d) All of the above

17. Consider the following statements :

1. A UMP test is unbiased.
2. A consistent test is UMP.
3. A UMP test is biased.
4. An unbiased test need not be UMP.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 and 3 only

(c) 1 and 2 only

(d) 1, 2 and 4

18. Let X_1, X_2 be iid random variables from $U(0, \theta)$. The power of the UMP test for testing $H_0 : \theta = 2$ against $H_1 : \theta > 2$ and $H_0 : \theta = 2$ against $H_1 : \theta \neq 2$ is

(a) $1 - \left(\frac{2}{\theta_1}\right)^2 (1 - \alpha)$

(b) α

(c) $1 - \left(\frac{\theta_1}{2}\right)^2 (1 - \alpha)$

(d) $1 - \alpha$

(Assume α is the size of the test and θ_1 is any value of θ under H_1)

19. Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from $N(\mu, 1)$ distribution. For testing $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1 > \mu_0$, the best critical region by NP lemma is

(a) $\bar{x} \geq \frac{1}{2}(\mu_0 + \mu_1) + \frac{\log k_\alpha}{n(\mu_1 - \mu_0)}$

(b) $\bar{x} \leq \frac{1}{2}(\mu_0 + \mu_1) + \frac{\log k_\alpha}{n(\mu_1 - \mu_0)}$

(c) $\bar{x} \geq \frac{1}{2}(\mu_1 - \mu_0) + \frac{\log k_\alpha}{n(\mu_1 - \mu_0)}$

(d) $\bar{x} \leq \frac{1}{2}(\mu_1 - \mu_0) + \frac{\log k_\alpha}{n(\mu_1 - \mu_0)}$

20. Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from an exponential distribution with pdf $f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, $x > 0$, $\theta > 0$. For testing the hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta = \theta_1$ $(< \theta_0)$, the best critical region of size α is

(a) $\left\{\mathbf{x} \mid \sum x_i > \frac{\theta_0}{2} \chi^2_{2n, \alpha}\right\}$

(b) $\left\{\mathbf{x} \mid \sum x_i < \frac{\theta_0}{2} \chi^2_{2n, \alpha}\right\}$

(c) $\left\{\mathbf{x} \mid \sum x_i > \frac{\theta_0}{2} \chi^2_{2n-1, \alpha}\right\}$

(d) None of the above

21. In general linear regression model $y = X\beta + u$; $u \sim N(0, \sigma^2 I)$, e is an $n \times 1$ residual vector and $\hat{y}$ is the estimated value of y. Define $H = X(X'X)^{-1}X'$. Then which of the following statements are correct ?

1. $V(e) = V(y) + V(\hat{y})$ where V(.) represents var-cov matrix
2. $Cov(y, \hat{y}) = \sigma^2 H$
3. $V(e) = \sigma^2(I - H)$

Select the correct answer using the code given below :

(a) 1 and 2 only
(b) 1 and 3 only
(c) 2 and 3 only
(d) 1, 2 and 3

22. For the Gauss Markov model $y = X\beta + u$; $V(u) = \sigma^2 I_n$. Let rank (X) = r and $l_i'\beta$ are m linearly independent estimable functions of β such that $l_i'\beta = \zeta_i$; i = 1, 2 ..., m, where l_i and ζ_i are known. Define RSS_1 as the residual sum of squares subject to the conditions $l_i'\beta = \zeta_i$; i = 1, 2, ..., m and RSS as the unconditional residual sum of squares. Then which of the following statements are correct ?

1. $(RSS_1 - RSS)/\sigma^2$ is distributed as a non-central chi square with m degrees of freedom.
2. RSS_1/RSS is distributed as central F with m and (n − r) degrees of freedom.
3. $(RSS_1 - RSS)$ and RSS are independently distributed.

Select the correct answer using the code given below :

(a) 1 and 2 only
(b) 1 and 3 only
(c) 2 and 3 only
(d) 1, 2 and 3

23. If A^- is any g-inverse of A and $A^-A = H$, then consider the following statements :

1. H is idempotent
2. AH = A
3. rank(A) = rank(H) = trace(H)

Which of the above statements are correct ?

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

24. Which of the following are the forms of the central limit theorem ?

1. Lindeberg-Feller theorem
2. Lyapunov Theorem
3. Khinchin Theorem

Select the correct answer using the code given below :

(a) 1 only
(b) 1 and 2
(c) 1 and 3
(d) 2 and 3

25. For the linear model $y = X\beta + u$; $E(u) = 0$, $V(u) = \sigma^2 I$, $\hat{\beta} = (X'X)^{-1}X'y$. Which of the following statements are correct ?

1. $V(\hat{\beta}) = \sigma^2(X'X)^{-1}$.
2. $\hat{\beta}$ is the BLUE for β.
3. Residual mean square is an unbiased estimator of σ^2.

Select the correct answer using the code given below :

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

26. Suppose $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ be n observations on certain geographical region, related through the model $y_i = \alpha + \beta x_i + u_i$, $i = 1, 2, ..., n$ where u_i's are the random errors and both x_i and y_i are measured in square metres. Let $\hat{\alpha}$ and $\hat{\beta}$ be the least squares estimators of α and β respectively. If the results are to be reported in square feet then the values of

(a) $\hat{\alpha}$ may change but $\hat{\beta}$ will not change

(b) $\hat{\beta}$ may change but $\hat{\alpha}$ will not change

(c) both $\hat{\alpha}$ and $\hat{\beta}$ will change

(d) both $\hat{\alpha}$ and $\hat{\beta}$ will not change

27. For the model $y = X\beta + u$ where y is an $n \times 1$ vector, X is an $n \times p$ matrix, β is an $n \times 1$ vector of unknown coefficients and u is the disturbance vector, if $\hat{\beta}$ is the solution of the equation $X'X\hat{\beta} = X'y$, then which of the following statements is/are correct ?

1. If $C'\beta$ is estimable, then $C'\hat{\beta}$ is the BLUE.
2. All linear parametric functions are estimable iff Rank(X) > p.
3. The hypothesis $H_0 : L\beta = \zeta$ where L is an $r \times p$ matrix and ζ is an $r \times 1$ vector of known constants, may be tested using Snedecor's F statistic.

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) 1 and 3

(d) 2 and 3

28. For 5 observations from a two variable linear regression model, it is observed that $\Sigma x_i = 0$, $\Sigma y_i = 5$, $\Sigma x_i y_i = 12$, $\Sigma x_i^2 = 10$, $\Sigma y_i^2 = 20$. The estimated error variance from the above observations, for the model is

(a) 0·1

(b) 0·2

(c) 0·3

(d) 1·0

29. Consider the model $E(y_1) = 2\beta_1 + \beta_2$, $E(y_2) = \beta_1 - \beta_2$ and $E(y_3) = \beta_1 + \alpha\beta_2$ with usual assumptions of linear models. The value of α such that the best linear unbiased estimators of β_1 and β_2 are uncorrelated, is

(a) – 1

(b) + 1

(c) 0

(d) + 2

30. For Gauss-Markov Linear (GML) model, $E(y_i) = \alpha_{i1}\beta_1 + \alpha_{i2}\beta_2 + ... + \alpha_{ip}\beta_p$ where $i = 1, 2, 3, ... n$; $V(y_i) = \sigma^2$ and $cov(y_i, y_j) = 0 \ (i \neq j)$

Consider the following statements :

1. If $\alpha_1, \alpha_2, ..., \alpha_p$, are linearly independent $(p \leq n)$, then GML model will be of full rank.
2. If all the parameters β's are fixed constants, then GML model is called a fixed effect model.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

31. Let $X_1, X_2, X_3, ..., X_n$ be iid $N(\mu, \sigma^2)$ and $\hat{\mu}$ and $\hat{\sigma}^2$ are the mle of μ and σ^2 respectively. Consider the following statements :

1. $\hat{\mu}$ is an unbiased estimator of μ.
2. $\hat{\sigma}^2$ is an unbiased estimator of σ^2.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

32. Let L be the likelihood function differentiable at least twice and $\theta = \hat{\theta}$ be a solution of the equation $\frac{\partial(\log L)}{\partial\theta} = 0$. If $l = \left[\frac{\partial^2 \log L}{\partial\theta^2}\right]_{\theta=\hat{\theta}}$, then $\hat{\theta}$ is a maximum likelihood estimate of θ if

(a) $l < 0$

(b) $l > 0$

(c) $l = 0$

(d) $l \geq 0$

33. Let X_1, X_2, X_3 be iid random variables with $U(\theta, \theta^2)$; $\theta > 1$. Then maximum likelihood estimator (mle) of θ is

(a) $X_{(1)}^2$

(b) $\sqrt{X_{(3)}}$

(c) $\sqrt{X_{(1)}}$

(d) $\alpha X_{(1)} + (1 - \alpha) X_{(3)}$

34. If t is an unbiased estimator of θ, then the minimum variance bound is attained if

(a) $\frac{\partial^2 \log L}{\partial\theta^2} = \frac{t - \theta}{\lambda}$

(b) $\frac{\partial \log L}{\partial\theta} = \frac{t}{\theta}$

(c) $\frac{\partial^2 \log L}{\partial\theta^2} = \frac{t - \theta}{\theta}$

(d) $\frac{\partial \log L}{\partial\theta} = \frac{t - \theta}{\lambda}$

(where λ is free from sample values but may depend on θ)

35. The estimator whose variance attains the Cramer – Rao lower bound for estimating the variance of a normal distribution with mean zero is

(a) $\frac{1}{n-1}\sum(X_i - \bar{X})^2$

(b) $\frac{1}{n}\sum(X_i - \bar{X})^2$

(c) $\frac{1}{n}\sum X_i^2$

(d) $\frac{1}{n-1}\sum X_i^2$

36. Let $X_1, X_2, X_3, ..., X_n$ be iid random variables with U(0, θ). The CR lower bound for any unbiased estimator of θ

(a) does not exist

(b) is $X_{(n)}$

(c) is $\frac{\theta}{n}$

(d) is $\frac{1}{\theta}$

37. Let $x_1, x_2, x_3, ..., x_n$ be a random sample from a population with pdf

$$f(x, \theta) = \begin{cases} \frac{1}{\theta}\exp\left(-\frac{x}{\theta}\right); 0 < x < \infty \\ 0; \text{otherwise} \end{cases}$$

The Cramer – Rao lower bound to the variance of an unbiased estimator of θ is

(a) $\frac{\theta}{n}$

(b) θ^2

(c) $\frac{\theta^2}{n}$

(d) θ

38. Let X_1, X_2 be iid random variables with N(θ, 1). Then $E(\overline{X} \mid X_1)$, where $\overline{X} = \frac{x_1 + x_2}{2}$, is

(a) X_1

(b) θ

(c) $X_1 + \theta$

(d) $\frac{X_1 + \theta}{2}$

39. Let (X_1, X_2) denote a random sample of size 2 from a binomial distribution with parameters 2 and θ, 0 < θ < 1. The sufficient statistic for θ is given by

(a) $X_1 - X_2$

(b) $X_1 + X_2$

(c) $\frac{X_1}{X_2}$

(d) $\frac{X_2}{X_1}$

40. The minimum variance unbiased estimator of θ^2 based on a sample of size n from N(θ, 1) is

(a) $\overline{X}^2 - \frac{1}{n}$

(b) $\overline{X}^2 + \frac{1}{n}$

(c) $\frac{\Sigma(X_i - \overline{X})^2}{n}$

(d) $\frac{\Sigma(X_i - \overline{X})^2}{n-1}$

41. If $X_1, X_2, X_3, ..., X_n$ are iid random variables with $N(\theta, \theta^2)$ then $E\left(\frac{n\overline{X}^2}{n+1} - S^2\right)$, where

$S^2 = \frac{\Sigma(X_i - \overline{X})^2}{n-1}$, is equal to

(a) 0

(b) θ

(c) θ^2

(d) $\frac{\theta}{n}$

42. The family of distributions $N(0, \sigma^2)$ is
1. Symmetric
2. Complete

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

43. If T_1 is a sufficient statistic of θ and T_2 is an unbiased estimator of θ, then an improved estimator of θ in terms of its efficiency is

(a) $E(T_1T_2)$
(b) $E(T_1 + T_2)$
(c) $E(T_1 \mid T_2)$
(d) $E(T_2 \mid T_1)$

44. If, for a given α, $0 \le \alpha \le 1$, non-randomised Neyman-Pearson and likelihood ratio test of a simple hypothesis against a simple alternative exists, then which one of the following is correct ?

(a) They are one and the same

(b) They are equivalent

(c) They are exactly opposite

(d) One cannot say anything about it

45. Let $X_1, X_2, X_3, ..., X_n$ be iid with density $f(x, \mu) = e^{-(x-\mu)}$, $x \ge \mu$. The MLE of $P(X_1 \ge t)$ for $t > \mu$ is given by

(a) $e^{-(t-\overline{x})}$

(b) $e^{-(t-X_{(1)})}$ where $X_{(1)} = \min(X_1, X_2, X_3, ..., X_n)$

(c) $e^{-(t-X_{(n)})}$ where $X_{(n)} = \max(X_1, X_2, X_3, ..., X_n)$

(d) $e^{-\left(t - \left(\frac{X_{(1)} + X_{(n)}}{2}\right)\right)}$

46. If for an estimator t_n of the parameter θ, $\lim_{n\to\infty} P\{|t_n - \theta| < \varepsilon\} = 1$ for all $\varepsilon > 0$, then

(a) t_n is a consistent estimator of θ

(b) t_n is an unbiased estimator of θ

(c) t_n is a sufficient estimator of θ

(d) t_n is an efficient estimator of θ

47. Let X be a Bernoulli random variable with parameter p and let $h(p) = p(1-p)$. Then maximum likelihood estimator of h(p), based on a sample of size n, is

(a) $\sum_{i=1}^{n} X_i\left(1 - \sum_{i=1}^{n} X_i\right)$

(b) $\frac{\overline{X}^2}{n^2}$

(c) $\overline{X}(1 - \overline{X})$

(d) $\frac{\overline{X}}{n}\left(1 - \frac{\overline{X}}{n}\right)$

48. If S(X) is a complete sufficient statistic and T(X) is ancillary statistic, then which one of the following statements is correct ?

(a) S(X) and T(X) are distributionally dependent

(b) S(X) and T(X) are functionally dependent

(c) S(X) and T(X) are statistically independent

(d) None of the above

49. A statistic T(x) for θ is said to be ancillary if

(a) T(x) is independent of θ

(b) T(x) is dependent on θ

(c) The distribution of T(x) is independent of θ

(d) The distribution of T(x) depends on θ

50. Suppose T^* is the most efficient estimate of θ and T is another estimate whose efficiency is e. Then the correlation coefficient between T^* and T will be

(a) $-e$

(b) e^2

(c) $e^{1/2}$

(d) e

51. Which of the following provides a quick snapshot of manufacturing sector performance in India ?

(a) Wholesale Price Index

(b) Annual Survey of Industries

(c) Index of Industrial Production

(d) Wage Rate Index

52. In NSSO surveys, a person would be considered as "not in labour force" if he/she

1. is not working.
2. is not seeking work.
3. is not available for work.
4. had a minor accident and so could not go to work for last 7 days.

Select the correct answer using the code given below :

(a) 4 only

(b) 1 and 2 only

(c) 1, 2, 3 and 4

(d) 1, 2 and 3 only

53. Which of the following are correct regarding Sustainable Development Goals (SDGs) endorsed by UNDP ?

1. SDGs came into effect in January 2016.
2. SDGs are adopted by more than 150 countries.
3. SDGs are mandatory to follow by member countries.
4. SDGs are 17 in number.

Select the correct answer using the code given below :

(a) 1, 2 and 3

(b) 2, 3 and 4

(c) 1, 2 and 4

(d) 1, 3 and 4

54. Which among the following is ***not*** an activity of CSO ?

(a) Compilation of National Accounts

(b) Conducting Economic Census

(c) Conducting Consumer Expenditure Survey

(d) Training of Statistical Personnel

55. Which of the following is ***not*** a function of National Statistical Commission (NSC) ?

(a) To conduct NSSO surveys

(b) To evolve national policies and priorities relating to the statistical system

(c) To evolve standard statistical concepts

(d) To identify the core statistics which are of national importance

56. The principal source of Industrial Statistics in India is

(a) Index of Industrial Production (IIP)

(b) Annual Survey of Industries (ASI)

(c) Economic Census

(d) Micro, Small and Medium Enterprises (MSME) Census

57. National income estimation is the responsibility of

(a) NSSO

(b) CSO

(c) Finance Ministry

(d) National Income Committee

58. Which of the following activities are undertaken by the office of Registrar General of India ?

1. Conducting population census in India
2. Conducting Sampling Registration System in India
3. Preparing and maintaining National Register of Indian Citizens (NRIC)
4. Conducting National Sample Survey Organization (NSSO) surveys
5. Conducting Economic Census

Select the correct answer using the code given below :

(a) 1, 2 and 5 only

(b) 1, 2 and 3 only

(c) 1, 2, 3, 4 and 5

(d) 3, 4 and 5 only

59. Indian statistical system is

1. Centralized
2. Laterally decentralized
3. Vertically decentralized

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) 3 only

(d) 2 and 3

60. Nodal Ministry of Statistical system in India is

(a) Ministry of Human Resource Development

(b) Ministry of Statistics and Programme Implementation

(c) Directorate of Economics and Statistics, Ministry of Agriculture

(d) None of the above

61. A test in which probability of rejecting H_0 when it is not true is more than that of rejecting it when it is true, is said to be

(a) Unbiased

(b) Biased

(c) Consistent

(d) Uniformly most powerful

62. Let X be a single observation from f(x). To test $H_0 : f(x) = f_0(x)$ for all x against $H_1 : f(x) = f_1(x)$ for all x, consider the test with test function

$$\phi(x) = \begin{cases} 1, & \text{if } f_1(x) > kf_0(x) \\ \gamma, & \text{if } f_1(x) = kf_0(x) \\ 0, & \text{if } f_1(x) < kf_0(x) \end{cases}$$

Suppose that $\gamma > 0$, then which one of the following is correct ?

(a) $\phi(x)$ is a non-randomised test function

(b) $\phi(x)$ is neither a non-randomised nor a randomised test function

(c) $\phi(x)$ is a randomised test function

(d) None of the above

63. Let $X_1, X_2, X_3, ..., X_n$ be iid random variables following $N(\theta, 1)$. The uniformly most powerful unbiased test (UMPU test) for testing $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta \neq \theta_0$ of size α, is

(a) $\phi(x) =$

$$\begin{cases} 1; & \overline{X} < \theta_0 - \frac{Z_{\frac{\alpha}{2}}}{\sqrt{n}} \text{ or } \overline{X} > \theta_0 + \frac{Z_{\frac{\alpha}{2}}}{\sqrt{n}} \\ 0; & \text{otherwise} \end{cases}$$

(b) $\phi(x) =$

$$\begin{cases} 1; & \overline{X} < \theta_0 - \frac{Z_{\alpha}}{\sqrt{n}} \text{ or } \overline{X} > \theta_0 + \frac{Z_{1-\alpha}}{\sqrt{n}} \\ 0; & \text{otherwise} \end{cases}$$

(c) $\phi(x) =$

$$\begin{cases} 1; & \overline{X} < \theta_0 - \frac{Z_{\alpha}}{n} \text{ or } \overline{X} > \theta_0 + \frac{Z_{1-\alpha}}{n} \\ 0; & \text{otherwise} \end{cases}$$

(d) $\phi(x) =$

$$\begin{cases} 1; & \overline{X} < \theta_0 - \frac{Z_{\frac{\alpha}{2}}}{n} \text{ or } \overline{X} > \theta_0 + \frac{Z_{\frac{\alpha}{2}}}{n} \\ 0; & \text{otherwise} \end{cases}$$

64. Let X have the distribution $f(x, \theta) = \theta^x(1-\theta)^{1-x}$; x = 0, 1; $0 < \theta < 1$. For testing $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta = \theta_1$, if $Z = \log\left\{\frac{f(x, \theta_1)}{f(x, \theta_0)}\right\}$, then E(Z) is equal to

(a) $\theta \log\left\{\frac{\theta_1(1-\theta_0)}{\theta_0(1-\theta_1)}\right\}$

(b) $\theta \log\left\{\frac{1-\theta_1}{1-\theta_0}\right\}$

(c) $\theta \log\left\{\frac{\theta_1(1-\theta_0)}{\theta_0(1-\theta_1)}\right\} + \log\left\{\frac{1-\theta_1}{1-\theta_0}\right\}$

(d) $\theta \log\left\{\frac{\theta_1(1-\theta_0)}{\theta_0(1-\theta_1)}\right\} + \theta \log\left\{\frac{1-\theta_1}{1-\theta_0}\right\}$

65. Two samples of size 15 were selected from $N(\mu, \sigma^2)$. The degrees of freedom for testing the difference between two population means will be

(a) 14
(b) 28
(c) 29
(d) 30

66. If $\overline{X}$ is the mean of a random sample of size n from a distribution with mean μ and variance 1, then an unbiased estimator for μ^2 is

(a) $\overline{X}^2$

(b) $\overline{X}^2 + \frac{1}{n}$

(c) $\overline{X}^2 - \frac{1}{n}$

(d) $\left(\overline{X} - \frac{1}{n}\right)^2$

67. Consider the following statements in respect of iid Poisson (λ) variates $x_1, x_2, x_3, ..., x_n$

1. $T = \Sigma x_i$ is complete sufficient for λ
2. $T = \Sigma x_i$ is the MLE of λ

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

68. Let (X_1, X_2) be iid random variables with N(θ, 1). Then number of unbiased estimators of θ is

(a) 1
(b) 3
(c) 5
(d) Infinite

69. Let $x_1, x_2, x_3, ..., x_n$ be a random sample of size n from Cauchy distribution with pdf $f(x, \theta) = \frac{1}{\pi[1 + (x-\theta)^2]}$, $-\infty < x < \infty$. Then the minimum variance bound estimator

(a) is Σx_i

(b) is $\overline{x}$

(c) is s^2

(d) does not exist

70. Let $x_1, x_2, x_3, ..., x_n$ be a random sample of size n from Bernoulli (p) distribution. The UMVUE of $\frac{1}{p}$

(a) is $\overline{x}$

(b) is $\frac{1}{\overline{x}}$

(c) is $\frac{1}{\Sigma x_i}$

(d) does not exist

71. Let x_1, x_2, x_3, ..., x_n be a random sample from a uniform population with pdf

$$f(x; \alpha, \beta) = \begin{cases} \frac{1}{\beta - \alpha}, & \alpha \le x \le \beta \\ 0, & \text{otherwise} \end{cases}$$

The sufficient statistic for the parameters

(a) will be x_1 and x_n

(b) will be $x_{(1)}$ and $x_{(n)}$

(c) will be $\overline{x}$ and s^2

(d) will not exist

72. If X_1, X_2 be iid $P(\lambda)$ random variables, then which one of the following is correct ?

(a) $X_1 + X_2$ is sufficient for λ

(b) $X_1 + 2X_2$ is sufficient for λ

(c) $X_1 + X_2$ is unbiased for λ

(d) $(X_1 + X_2)/2$ is not UMVUE for λ

73. The uniformly most powerful test for testing $H_0 : X \sim f_0(x, \theta)$ against $H_1 : X \sim f_1(x, \theta)$ if it exists, then

(a) it is unique

(b) it is unique but power is greater than size

(c) it need not be unique

(d) None of the above

74. The Bayes estimator of θ, when $X \sim f_\theta(x)$ and θ has prior distribution $\pi(\theta)$, for squared error loss function is

(a) Mean of the distribution, $f_\theta(x)$

(b) Mean of the prior distribution, $\pi(\theta)$

(c) Mean of the posterior distribution, $\pi(\theta | x)$

(d) Median of the posterior distribution, $\pi(\theta | x)$

75. Let x_1, x_2, x_3, ..., x_n be a sample of size n from a population which follows Poisson (λ) distribution. Which of the following statements is/are correct ?

1. Cramer – Rao bound to the variance of unbiased estimator of θ is $\frac{\theta}{n}$.

2. $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is a minimum vaiance bound unbiased estimator of θ.

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

76. Which one of the following divisions of CSO deals with Environment Statistics ?

(a) Economic Statistics Division

(b) National Accounts Division

(c) Social Statistics Division

(d) Price Statistics Division

77. Which of the following actions are correctly matched with the fundamental principles of Official Statistics ?

1. A media publishes a wrong interpretation of data released by the government and so the government brings out an article with the correct interpretation – Prevention of Misuse (Fundamental Principle 4).
2. Government frames Collection of Statistics Act – Sources of Official Statistics (Principle 5).
3. Unit level data is released and by principle of exclusion, an individual's data is publicly disclosed leading to violation of this fundamental principle – Confidentiality (Principle 6).

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

78. The subjects to be covered under 76th round of NSS are

1. Disability
2. Drinking water, sanitation, hygiene and housing conditions
3. Household consumption expenditure

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 1 and 3 only

(c) 2 and 3 only

(d) 1, 2 and 3

79. Minimum Support Price (MSP) is recommended by

(a) Directorate of Economics and Statistics

(b) Directorate of Marketing and Inspection

(c) Commission for Agricultural Costs and Prices (CACP)

(d) None of the above

80. Which of the below mentioned data sources can be used as a sampling frame for another survey ?

1. Population Census
2. National Family Health Survey
3. Situation Assessment Survey
4. Economic Census

Select the correct answer using the code given below :

(a) 2 and 3 only

(b) 1 and 4 only

(c) 4 only

(d) 1, 2 and 4

SPACE FOR ROUGH WORK

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