

## Indices and Surds

### Type - I

⇒ Let  $n$  be a positive integer and  $a$  be real number, then:

$$a^n = \frac{a \times a \times a \times \dots \times a}{(n \text{ factors})}$$

$a^n$  is called “ $n^{\text{th}}$  Power of  $a$ ” or

“ $a$  raised to the power  $n$ ”

where,  $a$  is called the base and  $n$  is called index or exponent of the power  $a^n$ .

**E.x.**  $3^2$  = square of 3,  $3^3$  = cube of 3 etc.

### Laws of Indices:

1.  $a^m \times a^n = a^{m+n}$  where  $a \neq 0$  and  $(m, n) \in I$

2.  $a^n \times a^n \times a^p \times \dots = a^{m+n+p+\dots}$

3.  $\frac{a^m}{a^n} = \begin{cases} a^{m-n} & \text{if } m > n \\ 1 & \text{if } n > m \\ \frac{1}{a^{n-m}} & \text{if } m = n \end{cases}$

4.  $(a^m)^n = a^{nm} = (a^n)^m$

5.  $a^{m^n} = a^{m \times m \times \dots \times m}$  times  $n$   $\neq (a^m)^n$

6.  $(ab)^n = a^n b^n$

7.  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

8.  $(-a)^n = \begin{cases} a^n, & \text{when } n \text{ is even} \\ -a^n, & \text{when } n \text{ is odd} \end{cases}$

These rules are also true when  $n$  is negative or fraction.

9.  $a^n = a^{(-1)n} = (a^{-1})^n = \left(\frac{1}{a}\right)^n$

$= \frac{1}{a} \times \frac{1}{a} \times \frac{1}{a} \dots \dots n$  times

10.  $a^{p/q} = a^{1/q \times p} = \left(a^{1/q}\right)^p$  is positive integer,  $q \neq 0$

$= a^{1/q} \times a^{1/q} \times \dots \dots p$  times

- $a^m = a^n \Rightarrow m = n$  when  $a \neq 0, 1$

- $a^m = b^m \Rightarrow a = b$

**Ex:**  $\left(-\frac{1}{343}\right)^{-\frac{2}{3}}$

**Sol.**  $\left(-\frac{1}{343}\right)^{-\frac{2}{3}} = \left(-\frac{1}{7^3}\right)^{-2/3} = (-7^{-3})^{-2/3}$

$= (-7)^{-3 \times -\frac{2}{3}} = (-7)^2 = 49$



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**Ex:**  $3^{-3} + (-3)^3$

**Sol.**  $3^{-3} + (-3)^3 = \frac{1}{3^3} + (-3)^3 = \frac{1}{27} - 27$   
 $= \frac{1-729}{27} = -\frac{728}{27}$

**Ex:** If  $2^{2x-1} = \frac{1}{8^{(x-3)}}$ , then  $x = ?$

**Sol.**  $2^{2x-1} = \frac{1}{8^{(x-3)}} \Rightarrow 2^{2x-1} = \frac{1}{2^{3(x-3)}}$

$\Rightarrow 2^{2x-1} = \frac{1}{2^{3x-9}}$

$\Rightarrow (2^{2x-1})(2^{3x-9}) = 1$

$\Rightarrow 2^{(2x-1)+(3x-9)} = 1$

$\Rightarrow 2^{5x-10} = 1 \Rightarrow 2^{5(x-2)} = 1$

$\Rightarrow 2^{5(x-2)} = 2^0$

$\Rightarrow x - 2 = 0 \Rightarrow x = 2$

### Type - II

**Surd:** If a rational and  $n$  is a positive integer and  $a^{1/n} = \sqrt[n]{a}$  is

Irrational, then  $\sqrt[n]{a}$  is called "surd of order  $n$ " or " $n^{\text{th}}$  root of  $a$ ". For the surd  $\sqrt[n]{a}$ ,  $n$  is called the surd - index or the order of the surd and " $a$ " is called the radicand. The symbol " $\sqrt{\quad}$ " is called the surd sign or radical.

**Ex.**  $\sqrt{5}$  is a surd of order 2 or square root of 5.

$\sqrt[3]{6}$  is surd of order 3 or cube root of 6.

$\sqrt{6+5}$  is not a surd as

$6 + \sqrt{5}$  is not a rational number.

- Every surd is an irrational number but every irrational number is not a surd.
- In the surd  $a\sqrt[n]{b}$ ,  $a$  and  $b$  are called factors of the surd.

**Ex.**  $3\sqrt{5}, 2\sqrt{7}, 5\sqrt[3]{7}$

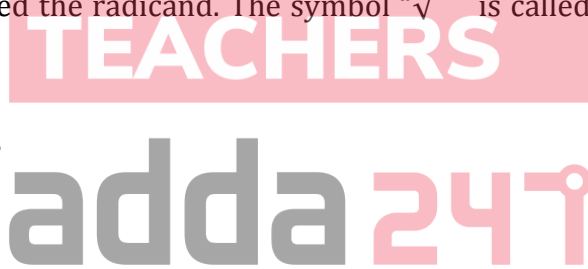
**Quadratic surd:** A Surd of order 2 (i.e.  $\sqrt{a}$ ) is called a quadratic surd.

**Ex:**  $\sqrt{2} = 2^{1/2}$  is a quadratic surd but  $\sqrt{4} = 4^{1/2}$  is not a quadratic surd because  $\sqrt{4} = 2$  is a rational number.

Therefore  $\sqrt{4}$  is not a surd.

**Cubic Surd:** A surd of order 3 (i.e.  $\sqrt[3]{a}$ ) is called a cubic surd.

**Ex.**  $\sqrt[3]{9}$  is a cubic surd but  $\sqrt[3]{27}$  is not a surd because  $\sqrt[3]{27} = 3$  is rational number.



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### Important Formula Based on Surds:

- (i)  $\sqrt[n]{a^n} = a$
- (ii)  $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- (iii)  $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$  and  $\frac{k\sqrt[n]{a}}{l\sqrt[n]{b}} = \frac{k}{l} \sqrt[n]{\frac{a}{b}}$
- (iv)  $\sqrt[m]{\sqrt[n]{a}} = \sqrt{mn}a = \sqrt[n]{\sqrt[m]{a}}$
- (v)  $(\sqrt[n]{a^m}) = (a)^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$
- (vi)  $\sqrt{a} \times \sqrt{a} = a$
- (vii)  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$  and  $k \cdot \sqrt[n]{a} \times l \cdot \sqrt[n]{b} = kl \cdot \sqrt[n]{a} \cdot \sqrt[n]{b} = kl \cdot \sqrt[n]{a^m b^n}$
- (viii)  $\sqrt{a^2 b} = a\sqrt{b}$
- (ix)  $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$
- (x)  $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$
- (xi)  $(\sqrt{a} + \sqrt{b}) \times (\sqrt{a} - \sqrt{b}) = a - b$ , where a and b are positive rational numbers.

**Ex.** The surd  $\sqrt[4]{3 \times 5^4}$  is not in its simplest form since the number under the radical sign had factor  $5^4$ . its index is equal to the order of the surd. Its simplest form:

$$\sqrt[4]{3 \times 5^4} = \sqrt[4]{3} \cdot \sqrt[4]{5^4} = (\sqrt[4]{3})(5) = 5 \cdot (\sqrt[4]{3})$$

**Similar or like Surds:** Surds having same irrational factors are called "similar or like surd".

**Ex.:**  $\sqrt[3]{3}$ ,  $7\sqrt{3}$ ,  $\frac{2}{5}\sqrt{3}$ ,  $\sqrt{3}$  etc. are similar surds.

**Unlike surds:** Surds have non - common irrational factors are called "unlike surds".

**Ex.**  $3\sqrt{3}$ ,  $5\sqrt{2}$ ,  $6\sqrt{7}$  etc. are unlike surds.

### Type III

**Comparison of Surds:** (i) If two surds are of the same order, then the one whose radicand is larger, is the larger of the two.

**Ex.**  $\sqrt[3]{19} > \sqrt[3]{15}$ ,  $\sqrt{7} > \sqrt{5}$ ,  $\sqrt[3]{9} > \sqrt[3]{7}$  etc.

(ii) If two surds are distinct order, we change them into the surds of the same order.

This order is L.C. M. of the orders of the given surds.

**Ex.:** Which is larger  $\sqrt{2}$  of  $\sqrt[3]{3}$  ?

**Sol.** Given surds are of order 2 & 3 respectively whose L.C.M is 6.

Convert each into a surd of order 6, as show below:

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$$\begin{aligned}\sqrt{2} &= 2^{\frac{1}{2}} = 2^{\frac{1}{2} \times \frac{3}{3}} = 2^{\frac{3}{6}} = (2^3)^{\frac{1}{6}} \\ &= (8)^{1/6} = \sqrt[6]{8} \\ &= 3^{\frac{1}{3}} = 3^{\frac{1}{3} \times \frac{2}{2}} = 3^{\frac{2}{6}} (9)^{\frac{1}{6}} \\ &= \sqrt[6]{9}\end{aligned}$$

Clearly,  $\sqrt[6]{9} > \sqrt[6]{8}$ , so  $\sqrt[3]{3} > \sqrt{2}$

**Type - IV**

(a) If  $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \dots \dots \infty}}}$

then,  $y = \frac{1 + \sqrt{1 + 4x}}{2}$

**Ex.:**  $y = \sqrt{7 + \sqrt{7 + \sqrt{7} \dots \dots \dots \infty}}$

**Sol.**  $y = \frac{1 + \sqrt{1 + 4x}}{2}$

Here,  $x = 7$

then  $y = \frac{1 + \sqrt{1 + 4 \times 7}}{2}$

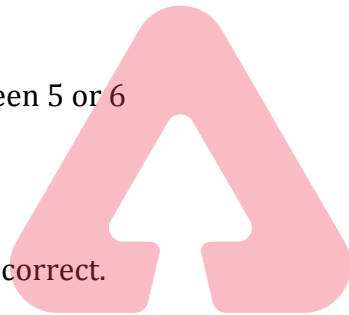
$= \frac{1 + \sqrt{29}}{2}$

$\therefore \sqrt{29}$  lies between 5 or 6

So,  $y = \frac{1+5}{2} = 3$

or,  $y = \frac{1+6}{2} = 3.5$

So,  $3 < y < 3.5$  is correct.



**Type - V**

Square - root of an irrational number:

As we know that,  $(a + b)^2 = (a^2 + b^2) + 2ab$

$\therefore (\sqrt{2} + \sqrt{3})^2 = \underbrace{5}_{(a^2+b^2)} + \underbrace{2\sqrt{6}}_{(2ab)}$

$\therefore 5 + 2\sqrt{6} = \underbrace{5 + 2\sqrt{2}\sqrt{3}}_{(2ab)}$

$\therefore a = \sqrt{2} \text{ \& } b = \sqrt{3}$

$\& a^2 + b^2 = 5$

$\therefore 5 + 2\sqrt{6} = (\sqrt{2} + \sqrt{3})^2$

$\Rightarrow a + b = \sqrt{5 + 2\sqrt{6}} = \sqrt{2} + \sqrt{3}$



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