Linear Equations

Linear Equations in Two variables: An equation of the form $ax + by + c = 0$ where $a, b, c \in R$ (real numbers) and $a \neq 0$, $b \neq 0$ and x, y are variables is called linear equation in two variables.

Examples: Each of the following equations is a linear equation:
(i) $4x + 7y - 13$
(ii) $2x - Sy = 36$
(iii) $\sqrt{3}x - \sqrt{7}y = 2$

Ex. One number is thrice the other number. When the large number is Subtracted from 49, the result is two more than the smaller number subtracted from 43. Find the numbers
Sol. Let a number = x
then second number = 3x
$49 - 3x = (43 - x) + 2$
$49 - 3x = 45 - x$
$4 = 2x$
x = 2, Number $\Longrightarrow$ 2, 6
The condition $a \neq 0$, $b \neq 0$, is often denoted by $a^2 + b^2 \neq 0$
The graph of a linear equation $ax + by + c = 0$, is a straight line

Solution of linear equation: Any pair of values of x and y which satisfy the equation $ax + by + c = 0$, is called its solution.

Ex. show that x = 2 and y = 1 is a solution of $2x + 5y = 9$
Sol: Substituting x = 2 and y = 1 in the given equation, we get RHS
$= 2 \times 2 + 5 \times 1 = 9$
$\therefore$ x =2, y = 1 is a solution of $2x + 5y = 9$

Ex. If $9x - 11 = -2x + 52$, then find the value of x.
Sol. $9x - 11 = -2x + 52$
$11x = 63$
$x = \frac{63}{11} \Longrightarrow 5\frac{8}{11}$

System of Linear Equations:
Consistent System: A system consisting of two simultaneous, linear equations is said to be consistent, if it has at least one solution.

Inconsistent System: A system consisting simultaneous linear equations is said to be inconsistent, if It has no solution at all.

Ex. Consider the system of equations: x + y = 9 & 3x + 3y = 5. Clearly, there are no values of x and y which may simultaneously satisfy the given equations. So, the system given above is inconsistent.

Conditions for Solvability: The system of equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ has:

(i) a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) an infinite number of solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_1}$

(iii) no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_1}$

Ex. The solutions of the equations $\frac{2x-y+2}{4} = \frac{3x+2y+3}{6} = \frac{4x+3y+1}{5}$

Sol. Solve by options (d)

$\frac{2\times1-0+2}{4} = \frac{3\times1+0+3}{6} = \frac{4\times1+0+1}{5}$

= 1 =1 = 1

So, x = 1, y = 0

Homogenous System of Equations: The system of equations $a_1e + b_1y = 0$; $a_2r + b_2y = 0$ has

(i) only solution x = 0, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) an infinite number of solutions when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

The graphs of $a_1x + b_1y + c_1 = 0$, $a_2x\ b_2y + c_2 = 0$ will be:

(i)parallel, if the system has no Solution;

(ii) coincident, if the system has infinite number of solutions;

(iii) Intersecting, if the system has a unique solution.

Ex. The value of x – y in the solution of the equations $\frac{x}{4} - \frac{y}{5} = 2$ and $\frac{x}{2} + y = 4$

Sol. $\frac{5x-4y}{20} = 2$

5x – 4y = 40 -------(1)×1

x + 2y = 8 ----------(2)×5

5x – 4y = 40

5x + 10y = 40

y = 0

x = 8

x – y = 8 – 0 ⟹ 8