SET~3

प्रश्न–पत्र कोड
Q.P. Code **65/3/3**

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गत

MATICS

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अधिकतम अंक : 80
Maximum Marks : 80

न पृष्ठ **23** हैं ।
ins **23** printed pages.

न प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains 38 questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a} \times \vec{b}| = 1$, then $\vec{a} \cdot \vec{b}$ is equal to

   (a) -1 (b) 1

   (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

2. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is :

   (a) $\frac{\pi}{2}$ (b) π

   (c) $\frac{\pi}{4}$ (d) 0

3. In Δ ABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to :

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

4. The equation of a line passing through point (2, – 1, 0) and parallel to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{2-z}{2}$ is :

(a) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{2}$ (b) $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{2}$

(c) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{-2}$ (d) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$

5. X and Y are independent events such that $P(X \cap \bar{Y}) = \frac{2}{5}$ and $P(X) = \frac{3}{5}$. Then P(Y) is equal to :

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) $\frac{1}{3}$ (d) $\frac{1}{5}$

6. The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is :

(a) 1 (b) 2

(c) any real number (d) 0

7. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is :

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

8. The number of feasible solutions of the linear programming problem given as

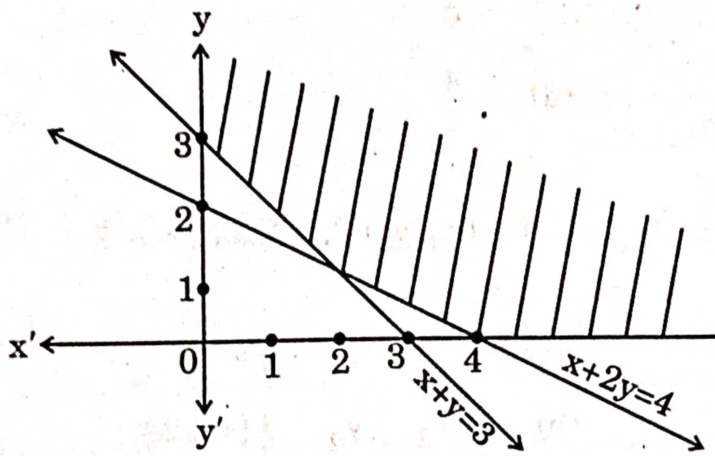
Maximize $z = 15x + 30y$ subject to constraints :

$3x + y \leq 12,\ x + 2y \leq 10,\ x \geq 0, y \geq 0$ is

(a) 1 (b) 2

(c) 3 (d) infinite

9. The feasible region of a linear programming problem is shown in the figure below :



Which of the following are the possible constraints ?

(a) $x + 2y \geq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(b) $x + 2y \leq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(c) $x + 2y \geq 4,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$

(d) $x + 2y \geq 4,\ x + y \geq 3,\ x \leq 0,\ y \leq 0$

10. A and B are square matrices of same order. If $(A + B)^2 = A^2 + B^2$, then :

(a) $AB = BA$ (b) $AB = -BA$

(c) $AB = O$ (d) $BA = O$

11. If $A . (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to :

(a) 12 (b) 9

(c) 3 (d) 27

12. A and B are skew-symmetric matrices of same order. AB is symmetric, if :

(a) $AB = O$ (b) $AB = -BA$

(c) $AB = BA$ (d) $BA = O$

13. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}\, I$, where

$$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} ?$$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) 0 (d) $\frac{\pi}{2}$

14. Let A be the area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3). Which of the following is correct ?

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$ (b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

(c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$ (d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

15. $\int 2^{x+2} dx$ is equal to :

(a) $2^{x+2} + C$ (b) $2^{x+2} \log 2 + C$

(c) $\frac{2^{x+2}}{\log 2} + C$ (d) $2 \cdot \frac{2^x}{\log 2} + C$

16. $\int e^{-x}\left(\frac{x+1}{x^2}\right) dx$ is equal to :

(a) $\frac{e^{-x}}{x}+C$

(b) $\frac{e^{x}}{x}+C$

(c) $\frac{e^{x}}{x^2}+C$

(d) $-\frac{e^{-x}}{x}+C$

17. The value of $\int_0^{\pi/2} \log \tan x \, dx$ is :

(a) $\frac{\pi}{2}$

(b) 0

(c) $-\frac{\pi}{2}$

(d) 1

18. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y+\left(\frac{dy}{dx}\right)^3\cos y=\sqrt{y}$?

(a) 3

(b) 2

(c) 6

(d) not defined

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

20. *Assertion (A) :* A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5).

Reason (R): Lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at x = 1.

OR

(b) If $x = a \sin 2t$, $y = a(\cos 2t + \log \tan t)$, then find $\frac{dy}{dx}$.

22. If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$.

23. Find the direction cosines of the line whose Cartesian equations are $5x - 3 = 15y + 7 = 3 - 10z$.

24. Find the points on the curve $6y = x^3 + 2$ at which ordinate is changing 8 times as fast as abscissa.

25. (a) Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

OR

(b) Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

OR

(b) There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

27. (a) Find the general solution of the differential equation :

$$\frac{d}{dx}(xy^2) = 2y(1+x^2)$$

OR

(b) Solve the following differential equation :

$$xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

28. Evaluate :

$$\int_0^{\pi/4} \log(1+\tan x)\,dx$$

29. (a) Find :

$$\int \frac{\cos x}{\sin 3x}\,dx$$

OR

(b) Find :

$$\int x^2 \log(x^2+1)\,dx$$

30. Find :

$$\int_{1}^{4} \frac{1}{\sqrt{2x+1}-\sqrt{2x-1}}\, dx$$

31. Solve the following linear programming problem graphically :

Minimize $z = x + 2y$

subject to the constraints

$$2x + y \geq 3,$$
$$x + 2y \geq 6,$$
$$x \geq 0,$$
$$y \geq 0.$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines.

OR

(b) Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

33. Check whether a function $f : \mathbb{R} \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^2}$ is one-one and onto or not.

34. The area of the region bounded by the line $y = mx$ $(m > 0)$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

35. (a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2\,I = O$.

OR

(b) If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$3x + 5y = 11, \; 2x - 7y = -3.$

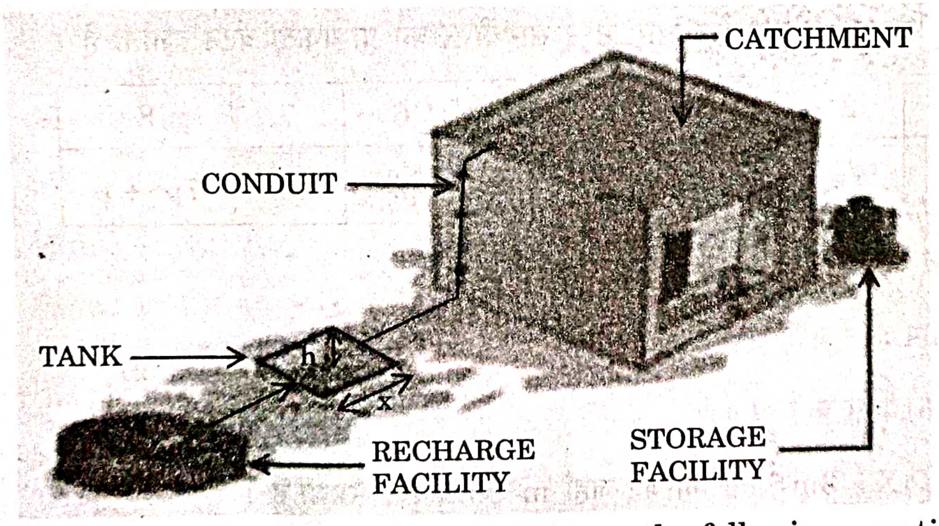
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x. *1*

(ii) Find $\frac{dC}{dx}$. *1*

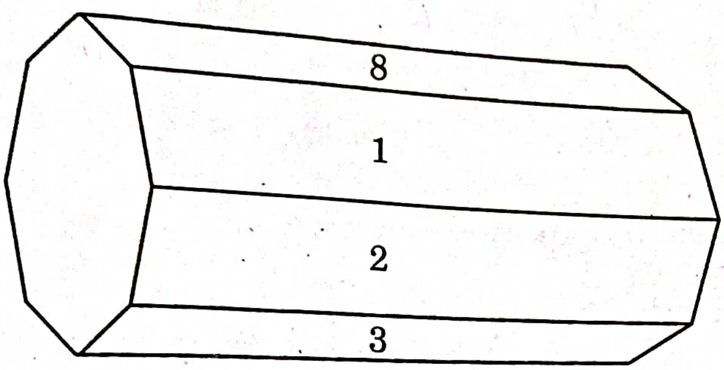
(iii) (a) Find the value of x for which cost C is minimum. 2

OR

(iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where x > 0. 2

Case Study – 2

37. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X) :	p	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following questions :

(i) Find the value of p. 1

(ii) Find P(X > 6). 1

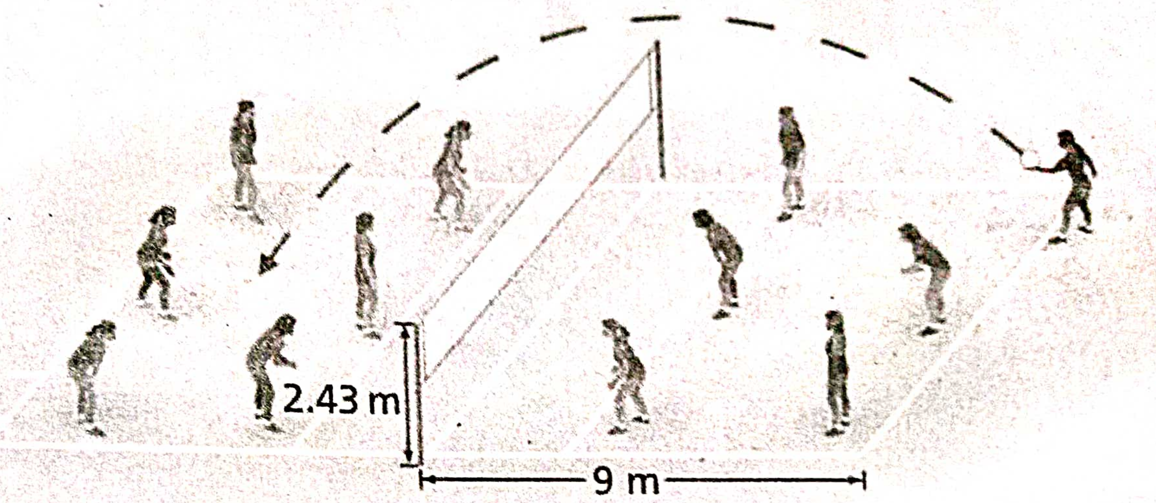
(iii) (a) Find P(X = 3m), where m is a natural number. 2

OR

(iii) (b) Find the mean E(X). 2

Case Study – 3

38. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.



Based on the above information, answer the following questions :

(i) Is h(t) a continuous function ? Justify. 2

(ii) Find the time at which the height of the ball is maximum. 2